Mock exam on Quantum Field Theory II

June 30, 2016
Duration: 2 hours

- Make sure that you write your name, your matriculation number **on each sheet of paper** that you hand in.
- Every solution to a problem should start on a new sheet.
- Write **clearly** if you want it to be corrected.

Good luck!

Obtained points:

Problem	1.	2.
Points	/60	/40

Problem 1: The pseudoscalar Yukawa theory (60 points)

Consider a real pseudoscalar field ϕ coupled to a Dirac fermion field ψ in $d = 4$ dimensions. The Lagrangian density of the model reads

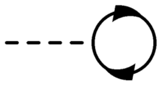
$$\mathcal{L}_I = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \bar{\psi}(i\partial\!\!\!/ - M)\psi - ig\bar{\psi}\gamma_5\psi\phi\,,$$

with g a real coupling constant and $\gamma_5 \equiv i\gamma_0\gamma_1\gamma_2\gamma_3$ the chirality matrix satisfying $\{\gamma_5, \gamma_\mu\} = 0$, $(\gamma_5)^\dagger = \gamma_5$ and $(\gamma_5)^2 = 1$.

1. Use functional techniques to determine the Feynmann rules of this theory in momentum space. Provide details about signs/line orientations related to fermions and possible symmetry factors. /10 pt.

2. Analyze the superficial degree of divergence of the amplitudes at one loop and beyond. /10 pt.

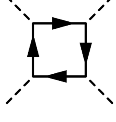
3. Show by explicit computation that the contribution of the tadpole diagram /10 pt.



is zero. Generalize this result by taking into account that the bilinear $\bar{\psi}\gamma_5\psi$ transforms as a pseudoscalar under parity transformations (i.e. $P\bar{\psi}(x)\gamma_5\psi(x)P^{-1} = -\bar{\psi}(\mathcal{P}x)\gamma_5\psi(\mathcal{P}x)$ with $\mathcal{P}(t,\vec{x}) = (t, -\vec{x})$). Use this fact, together with the transformation property of the scalar field ($P\phi(x)P^{-1} = -\phi(\mathcal{P}x)$) to argue that the one-point function and the three-point function

vanish at all orders in perturbation theory. *Hint*: For a massive particle you are always allowed to go to the rest frame.

4. Show by explicit computation that the diagram /10 pt.



diverges logaritmically.

5. Use the above results to argue which counterterms should be included when renormalizing the theory. Is the result consistent with the symmetries of the theory? Define a minimum set of renormalization conditions. /10 pt.

6. Write down explicitly the one-loop correction to the fermion propagator. Evaluate explicitly the divergent part using dimensional regularization (i.e. the coefficients of $1/(d-4)$ in the loop amplitude). The finite part of the amplitude does not need to be explicitly computed (i.e. work in the $\overline{\text{MS}}$ scheme.). /10 pt.

   Hints:

$$\frac{1}{AB} = \int_0^1 dx \frac{1}{[xA + (1-x)B]^2} = \int_0^1 dx\, dy\, \delta(x+y-1) \frac{1}{[xA + yB]^2}$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{[k^2 - \Delta^2]^2} = \frac{i}{(4\pi)^{d/2}} \Gamma(2 - d/2) \Delta^{d/2-2} .$$

$$\Gamma(\epsilon) = \frac{1}{\epsilon} + \mathcal{O}(1) \quad \text{around} \quad \epsilon = 0 .$$

Problem 2: YM theory in the Arnowitt-Fickler gauge (40 points)

1. How many degrees of freedom have the Faddev-Popov ghosts in QCD? And in a Yang-Mills theory with gauge group $SU(N)$? /10 pt.

2. Perform the Faddeev-Popov quantization of the Yang-Mills theory /10 pt.

$$\mathcal{L}_{YM} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \bar{\psi}(i\not{D} - m)\psi ,$$

   in the Arnowitt-Fickler gauge $A^{3a} = 0$. *Hint:* Write the gauge condition in a covariant way.

3. Write down the Feynmann rule for the gauge boson propagator in this gauge. /10 pt.

4. Show that in the $A^{3a} = 0$ gauge i) there are no propagating ghosts and that ii) the gauge condition is reduced to two positive metric degrees of freedom. /10 pt.